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Brief paper

Disturbance decoupled fault reconstruction using cascaded sliding mode observers[☆]

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ABSTRACT

This paper presents a disturbance decoupled fault reconstruction (DDFR) scheme using cascaded sliding mode observers (SMOs). The processed signals from a SMO are found to be the output of a fictitious system which treats the faults and disturbances as inputs; the 'outputs' are then fed into the next SMO. This process is repeated until the attainment of a fictitious system which satisfies the conditions that guarantee DDFR. It is found that this scheme is less restrictive and enables DDFR for a wider class of systems compared to previous work when only one or two SMOs were used. This paper also presents a systematic routine to check for the feasibility of the scheme and to calculate the required number of SMOs from the outset and also to design the DDFR scheme. A design example verifies its effectiveness.

1. Introduction

Fault reconstruction is an interesting and important area of research as it provides an estimate of faults in a system, so that accurate corrective action can be taken. However, most of these fault reconstruction schemes are designed about a model, which in reality is not a perfect representation of the system. The mismatches are considered as disturbances to the fault reconstruction scheme as they could raise false alarms or mask a fault, and potentially cause adverse consequences; hence the fault reconstruction should be insensitive to them. Tan and Edwards (2003) proposed a design method for a sliding mode observer (SMO) (Edwards & Spurgeon, 1994) such that the $\mathcal{L}_2$ gain from the disturbances to the fault reconstruction is minimized. Saif and Guan (1993) combined the faults and disturbances to form a new augmented 'fault' vector and reconstructed this new 'fault'. Although this successfully decouples the disturbances from the fault reconstruction, and achieves disturbance decoupled fault reconstruction (DDFR), it requires a set of very stringent conditions to be fulfilled and is conservative as the reconstruction of the disturbances is unnecessary; it is only necessary for the disturbances to be rejected/decoupled. Edwards and Tan (2006) then compared the performances of Edwards, Spurgeon, and Patton (2000) and Saif and Guan (1993), and found that disturbance reconstruction was not necessary in achieving DDFR, but did not show the conditions that guaranteed it. Ng, Tan, Akmeliawati, and Edwards (2010a) investigated the conditions that guarantee DDFR for the SMO, and found them less stringent than if a linear observer (Saif & Guan, 1993) was used. Following that, Ng, Tan, Man, and Akmeliawati (2010b) used 2 SMOs in cascade and showed that DDFR could be achieved for a wider class of systems compared to when only 1 SMO is used (Ng et al., 2010a).

This paper builds on the work in Ng et al. (2010b) and uses multiple SMOs in cascade to achieve DDFR. Using similar ideas from Ng et al. (2010b), signals from a SMO are processed and found to be the output of a 'fictitious' system that treats the faults and disturbances as inputs. Another observer is then designed for the fictitious system, and the scheme proposed in this paper does not stop at 2 observers as in Ng et al. (2010b); the steps of observing and processing signals to form a fictitious system are repeated iteratively until the attainment of a fictitious system that satisfies the DDFR conditions in Ng et al. (2010a). It is found that the scheme in this paper is applicable to a wider class of systems as compared to where only one or two observers are used (Ng et al., 2010a,b). From a physical/practical viewpoint, this could mean that the scheme in this paper could achieve DDFR with fewer number of output sensors compared to Ng et al. (2010a,b), hence results in a more compact system and reduces cost. The framework in this paper is accompanied by an

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outcome in the construction of a systematic method for calculating the number of observers required to achieve DDFR from the outset. This systematic calculation renders the technique (which involves repetitive design steps) highly practical, by providing the knowledge of the structure of the cascaded observer system prior to the design process. The design procedure of the components in the cascade observer scheme as well as a thorough analysis of the method are presented. An important result is that the sliding motion of an observer is *not affected* by the design of the previous SMOs. This means that the ability of a SMO to achieve DDFR is not affected by the design of the previous observers, significantly simplifying the design procedure.

The paper is organized as follows: Section 2 introduces the system and sets up the framework for the method in this paper, and presents the algorithm that calculates the number of SMOs that guarantee DDFR, followed by a design algorithm for the SMOs; Section 3 investigates the existence conditions for DDFR as well as the limitation of the scheme; Section 4 presents a simulation example. Finally, conclusions are summarized in Section 5.

2. Problem statement and design algorithm

Consider the system $\dot{x} = Ax + Mf + Q\xi, y = Cx$ where $x \in \mathbb{R}^n, y \in \mathbb{R}^p, f \in \mathbb{R}^q$ respectively are the states, outputs, faults and $\xi \in \mathbb{R}^h$ are disturbances that represent the mismatch between the linear model and the real plant. Chapter 5 of Chen and Patton (1999) elaborates on the appropriate selection of Q. Assume $p \geq q+h$. Without loss of generality, let $\mathrm{rank}(M) = q$, $\mathrm{rank}(C) = p$ and $\mathrm{rank}(CQ) = \bar{k} < h$. The objective is to generate a reconstruction of f decoupled from ξ, and hence achieve DDFR. Assume also $\mathrm{rank}(CM) = \mathrm{rank}(M)$, $\mathrm{rank}\left(C\begin{bmatrix} M & Q\end{bmatrix}\right) = \mathrm{rank}(CM) + \mathrm{rank}(CQ)$and hence from Proposition 1 in Ng et al. (2010a), there exist transformations for x and ξ such that A, Q, M, C can be written as:

$$A = \left[\begin{array}{cc|c} A_{11} & A_{12} & * \\ A_{13} & A_{14} & * \\ \hline A_{31} & A_{32} & * \\ A_{33} & A_{34} & * \\ A_{35} & A_{36} & * \end{array}\right],$$

$$Q = \left[\begin{array}{cc} 0 & 0 \\ Q_{11} & 0 \\ \hline 0 & 0 \\ 0 & Q_{22} \\ 0 & 0 \end{array}\right] \begin{array}{l} \updownarrow n-p-h+\bar{k} \\ \updownarrow h-\bar{k} \\ \updownarrow p-q-\bar{k} \\ \updownarrow \bar{k} \\ \updownarrow q \end{array} \tag{1}$$

$$C = \begin{bmatrix} 0 & C_2 \end{bmatrix}, \qquad M = \begin{bmatrix} 0 & M_o^T \end{bmatrix}^T \tag{2}$$

where Q_{11}, Q_{22}, C_2 and M_o are square and invertible; A_{11}, A_{14} are square, and $(*)$ are general matrices. In Ng et al. (2010a), DDFR can be achieved using a SMO if the following hold:

A1. $\mathrm{rank}\begin{bmatrix} A_{12}^T & A_{32}^T & A_{36}^T \end{bmatrix}^T = \mathrm{rank}(A_{32})$
A2. The zeros (if any) of $\left(A, \begin{bmatrix} M & Q \end{bmatrix}, C\right)$ are stable.

Ng et al. (2010b) used 2 cascaded SMOs to achieve DDFR when A1 was not satisfied. This paper seeks to achieve DDFR for cases where Ng et al. (2010a,b) are unable to achieve DDFR, by extending the work in Ng et al. (2010b) to go beyond 2 SMOs and thus enlarging the class of systems where DDFR can be achieved; systematic algorithms to calculate the required number of observers and to design the observers will also be presented.

For the notation in this paper, X^i indicates that the parameter X is evaluated for observer i. If X is raised to a power i, it shall be denoted by $(X)^i$.

2.1. Algorithm to calculate number of SMOs for DDFR

To achieve DDFR, A2 must hold; justifications for this is in Theorem 1. Assuming A2 is satisfied, define $\bar{k}^i := \bar{k}^{i-1} + k^i$, and also $A_{12}^1 := A_{12}, A_{31}^1 := A_{31}, A_{32}^1 := A_{32}, A_{36}^1 := A_{36}, Q_{11}^1 := Q_{11}, Q_{22}^1 := Q_{22}, n^1 := n, p^1 := p, \bar{k}^1 := \bar{k}, \bar{A} := A, \bar{Q} := Q, \bar{M} := M, \bar{C} := C$. Initialize the index variable $i = 1$ and enter the following iterative procedure:

(1) Define $k^{i+1} := \mathrm{rank}(A_{32}^i)$, $p^{i+1} := \mathrm{rank}\begin{bmatrix} A_{31}^i & A_{32}^i \end{bmatrix} + q + \bar{k}^i$. From (1) (and also (3)), $\begin{bmatrix} A_{31}^i & A_{32}^i \end{bmatrix}$ has a maximum rank of $p^i - q - \bar{k}^i$ since it has that number of rows; then it is obvious from the definition of p^{i+1} that $p^{i+1} \leq p^i$.

(2) Let $R_1^i \in \mathbb{R}^{(\breve{p}^i - \bar{p}^i)\times(\breve{p}^i - \bar{p}^i)}$, $R_4^i \in \mathbb{R}^{\bar{n}^i \times \bar{n}^i}$ and $R_3^i \in \mathbb{R}^{(h-\bar{k}^i)\times(h-\bar{k}^i)}$ and $R_{43}^i := \mathrm{diag}\left\{R_4^i, R_3^i\right\}$ such that

$$R_1^i \begin{bmatrix} A_{31}^i & A_{32}^i \end{bmatrix} R_{43}^i = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & A_{312}^i & 0 & 0 \\ A_{313}^i & A_{314}^i & 0 & A_{322}^i \end{bmatrix} \begin{array}{l} \updownarrow \breve{p}^i \\ \updownarrow \breve{p}^{i+1} - \bar{p}^{i+1} \\ \updownarrow k^{i+1} \end{array}$$

where $\breve{p}^i := p^i - p^{i+1}, \bar{p}^i = p^{i+1} - \bar{k}^i - q, \bar{n}^i := n^i - p^i - h + \bar{k}^i$ and A_{312}^i, A_{322}^i are square and invertible. Note that R_1^i can be computed by compressing the rows of A_{32}^i to the bottom k^{i+1}, followed by compressing the remaining rows of A_{31}^i to the next $\breve{p}^{i+1} - \bar{p}^{i+1}$. Matrix R_3^i can then be computed by compressing $R_1^i A_{32}^i$ to the last k^{i+1} columns, and R_4^i by compressing the top $\breve{p}^{i+1} - \bar{p}^{i+1}$ rows of $R_1^i A_{31}^i$ to the last $\breve{p}^{i+1} - \bar{p}^{i+1}$ columns. Define $T_\xi^i \in \mathbb{R}^{(h-\bar{k}^i)\times(h-\bar{k}^i)}$ where $(R_3^i)^{-1} Q_{11}^i (T_\xi^i)^{-1} = \mathrm{diag}\{Q_{11}^{i+1}, Q_{22}^{i+1}\}$ where $Q_{22}^{i+1} \in \mathbb{R}^{k^{i+1} \times k^{i+1}}$ is invertible.

(3) Define $T^i := T_3^i T_2^i T_1^i$ where T_1^i, T_2^i, T_3^i respectively are

$$\mathrm{diag}\{(R_4^i)^{-1}, (R_3^i)^{-1}, R_1^i, I_{\bar{k}^i}\}$$

$$\left[\begin{array}{ccc|c} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ (A_{322}^i)^{-1} A_{313}^i & 0 & I & 0 \\ \hline 0 & 0 & 0 & I_{\bar{p}^i} \end{array}\right],$$

$$\left[\begin{array}{cccc|c} I & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ \hline 0 & 0 & 0 & 0 & I \end{array}\right] \begin{array}{l} \updownarrow \bar{n}^i - \breve{p}^{i+1} - \bar{p}^{i+1} \\ \updownarrow h - \bar{k}^{i+1} \\ \updownarrow \breve{p}^{i+1} - \bar{p}^{i+1} \\ \updownarrow k^{i+1} \\ \updownarrow \bar{p}^i \end{array}$$

Define $\bar{T}^i = \mathrm{diag}\{T^i, I_{\Sigma_{j=1}^{i-2} \bar{p}^j}\}$, $\bar{T}_\xi^i = \mathrm{diag}\{T_\xi^i, I_{\bar{k}^i}\}$.

Then transform x, ξ so that $\bar{A} \mapsto \bar{T}^i \bar{A} (\bar{T}^i)^{-1}, \bar{Q} \mapsto \bar{T}^i \bar{Q} (\bar{T}_\xi^i)^{-1}$ with the respective structures:

$$\left[\begin{array}{cc|c|c|c|c} A_{11}^{i+1} & A_{12}^{i+1} & * & \cdots & * & * \\ A_{13}^{i+1} & A_{14}^{i+1} & * & \cdots & * & * \\ A_{31}^{i+1} & A_{32}^{i+1} & * & \cdots & * & * \\ A_{33}^{i+1} & A_{34}^{i+1} & * & \cdots & * & * \\ \hline 0 & 0 & 0 & \cdots & * & * \\ 0 & 0 & Y^i & \cdots & * & * \\ * & * & * & \cdots & * & * \\ \hline \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \hline 0 & 0 & 0 & \cdots & 0 & * \\ 0 & 0 & 0 & \cdots & Y^1 & * \\ * & * & * & \cdots & * & * \\ A_{35}^{i+1} & A_{36}^{i+1} & * & \cdots & * & * \end{array}\right] \begin{array}{l} \updownarrow n - h - \Sigma_{j=1}^{i+1}(\breve{p}^j + \bar{p}^j) \\ \updownarrow h - \bar{k}^{i+1} \\ \updownarrow \breve{p}^{i+1} + \bar{p}^{i+1} \\ \updownarrow k^{i+1} \\ \updownarrow \breve{p}^i \\ \updownarrow \bar{p}^i \\ \updownarrow k^i \\ \vdots \\ \updownarrow \breve{p}^1 \\ \updownarrow \bar{p}^1 \\ \updownarrow k^1 \\ \updownarrow q \end{array} \tag{3}$$

$$\begin{bmatrix} 0&0&0&\cdots&0\\ Q_{11}^{i+1}&0&0&\cdots&0\\ 0&\bar{Q}_{22}^{i+1}&0&\cdots&0\\ 0&0&\bar{Q}_{22}^{i}&\cdots&0\\ \vdots&\vdots&\vdots&\ddots&\vdots\\ 0&0&0&\cdots&\bar{Q}_{22}^{1}\\ 0&0&0&\cdots&0\end{bmatrix} \begin{array}{l} \updownarrow n-h-\Sigma_{j=1}^{i+1}(\breve{p}^j+\bar{p}^j)\\ \updownarrow h-\bar{k}^{i+1}\\ \updownarrow \breve{p}^{i+1}+\bar{p}^{i+1}+k^{i+1}\\ \updownarrow \breve{p}^i+\bar{p}^i+k^i\\ \vdots\\ \updownarrow \breve{p}^1+\bar{p}^1+k^1\\ \updownarrow q\end{array} \qquad (4)$$

where $Y^i := \begin{bmatrix} A_{312}^i & 0 \\ A_{314}^i & A_{322}^i \end{bmatrix}$, $\bar{Q}_{22}^i := \begin{bmatrix} 0 \\ Q_{22}^i \end{bmatrix}$ and $\det(Y^i) \neq 0$. The last $(*)$ column of $\bar{A}$ has p^1 columns, and $\bar{T}^i\bar{M}$, $\bar{C}(\bar{T}^i)^{-1}$ have structures identical to (2).

(4) (a) If $\text{rank}\begin{bmatrix}(A_{12}^{i+1})^T & (A_{32}^{i+1})^T & (A_{36}^{i+1})^T\end{bmatrix}^T = \text{rank}(A_{32}^{i+1})$, define $m = i+1$, exit this algorithm and proceed to the algorithm in Section 2.2 to design the SMOs. DDFR can now be achieved using m SMOs; the reason for this is in Step 3 of the design algorithm.

(b) If $\begin{bmatrix}A_{31}^{i+1} & A_{32}^{i+1}\end{bmatrix} = 0$ or does not exist, and if A_{36}^{i+1} or A_{12}^{i+1} are nonzero, then DDFR cannot be guaranteed and the algorithm is terminated. The reason for this is given in Proposition 1.

If neither (a) nor (b) are satisfied, define $n^{i+1} := n^i - p^i + \bar{k}^i + q$, increment i by 1 and return to Step 1.

2.2. Algorithm to design cascaded SMOs

Partition $\bar{A}$, $\bar{Q}$ as $\bar{A} = \begin{bmatrix} \bar{A}_1^i & \bar{A}_2^i \\ \bar{A}_3^i & \bar{A}_4^i \end{bmatrix}$, $\bar{Q} = \begin{bmatrix} \bar{Q}_1^i \\ \bar{Q}_2^i \end{bmatrix}$, where $\bar{A}_4^i$ is square, and $\bar{A}_3^i$, $\bar{Q}_2^i$ can then be partitioned further as:

$$\bar{A}_3^i = \begin{bmatrix} 0 \\ \bar{A}_{31}^i \\ \bar{A}_{32}^i \\ \bar{A}_{33}^i \\ \bar{A}_{34}^i \end{bmatrix}, \qquad \bar{Q}_2^i = \begin{bmatrix} 0 \\ 0 \\ \bar{Q}_{21}^i \\ \bar{Q}_{22}^i \\ 0 \end{bmatrix} \begin{array}{l} \updownarrow \breve{p}^i \\ \updownarrow \bar{p}^i \\ \updownarrow k^i \\ \updownarrow \Sigma_{j=1}^{i-1}\bar{p}^{j-2} \\ \updownarrow q \end{array} \qquad (5)$$

where the following can be extracted from (3):

$$\bar{A}_1^i = \left[\begin{array}{cc|c} A_{11}^{i+1} & A_{12}^{i+1} & * \\ A_{13}^{i+1} & A_{14}^{i+1} & * \\ \hline A_{31}^{i+1} & A_{32}^{i+1} & * \\ A_{33}^{i+1} & A_{34}^{i+1} & * \end{array}\right] \qquad \begin{array}{l} \bar{A}_{31}^i = \begin{bmatrix} 0 & Y^i \end{bmatrix} \\ \bar{Q}_{21}^i = \begin{bmatrix} 0 & Q_{22}^i & 0 \end{bmatrix} \\ \bar{A}_{34}^i = \begin{bmatrix} A_{35}^{i+1} & A_{36}^{i+1} & * \end{bmatrix}. \end{array} \qquad (6)$$

From (4), there are $\bar{k}^{i+1}$ columns to the right of Q_{22}^i in $\bar{Q}_{21}^i$ in (6) have $\bar{k}^{i-1}$ columns and there are $\bar{p}^i$ columns to the right of A_{32}^{i+1} in $\bar{A}_1^i$. Therefore

$$\bar{A}_1^i = \left[\begin{array}{c|c} \bar{A}_1^{i+1} & * \\ \hline 0 & * \\ \bar{A}_{31}^{i+1} & * \\ \bar{A}_{32}^{i+1} & * \end{array}\right], \qquad \bar{Q}_1^i = \left[\begin{array}{c} \bar{Q}_1^{i+1} \\ \hline 0 \\ 0 \\ \bar{Q}_{21}^{i+1} \end{array}\right] \begin{array}{l} \updownarrow n - \Sigma_{j=1}^{i}\bar{p}^j \\ \updownarrow \breve{p}^{i+1} \\ \updownarrow \bar{p}^{i+1} \\ \updownarrow k^{i+1}. \end{array} \qquad (7)$$

Define $\tilde{A}^1 := \bar{A}$, $\tilde{Q}^1 := \bar{Q}$, $\tilde{C}^1 := \bar{C}$, $\tilde{M}^1 := \bar{M}$, $x^1 := x$, $y^1 := y$, $M_o^1 := M_o$, set $i = 1$ and enter the following algorithm:

(1) *Implement the SMO*: The system under consideration with n^i states, p^i outputs, q faults and h disturbances is $\dot{x}^i = \tilde{A}^i x^i + \tilde{M}^i f + \tilde{Q}^i \xi$, $y^i = \tilde{C}^i x^i$ where $\tilde{C}^i = \begin{bmatrix} 0 & \tilde{C}_2^i \end{bmatrix}$ with $\tilde{C}_2^i \in \mathbb{R}^{p^i \times p^i}$ being invertible and

$$\tilde{A}^i = \begin{bmatrix} \tilde{A}_1^i & \tilde{A}_2^i \\ \tilde{A}_3^i & \tilde{A}_4^i \end{bmatrix}, \qquad \tilde{M}^i = \begin{bmatrix} 0 \\ \tilde{M}_2^i \end{bmatrix},$$

$$\tilde{Q}^i = \begin{bmatrix} \tilde{Q}_1^i \\ \tilde{Q}_2^i \end{bmatrix} \begin{array}{l} \updownarrow n^i - p^i \\ \updownarrow p^i \end{array} \qquad (8)$$

$$\tilde{A}_3^i = \begin{bmatrix} 0 \\ \tilde{A}_{31}^i \\ \tilde{A}_{32}^i \\ \tilde{A}_{33}^i \end{bmatrix}, \qquad \tilde{M}_2^i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ M_o^i \end{bmatrix}, \qquad \tilde{Q}_2^i = \begin{bmatrix} 0 \\ 0 \\ \tilde{Q}_{22}^i \\ 0 \end{bmatrix} \begin{array}{l} \updownarrow \breve{p}^i \\ \updownarrow \bar{p}^i \\ \updownarrow \bar{k}^i \\ \updownarrow q \end{array} \qquad (9)$$

where $\tilde{Q}_{22}^i = \begin{bmatrix} 0 & \tilde{Q}_{222}^i \end{bmatrix}$ with $\tilde{Q}_{222}^i$, M_o^i are invertible. Define $\bar{\alpha}^i = \alpha^i\alpha^{i-1}\cdots\alpha^2\alpha^1\alpha^0$ and $\alpha^0 = 1$. It can be seen that $\tilde{A}_{33}^i = \bar{\alpha}^{i-1}\bar{A}_{34}^i$, $\tilde{A}_{31}^i = \bar{A}_{31}^i$, $\tilde{A}_1^i = \bar{A}_1^i$, $\tilde{Q}_1^i = \bar{Q}_1^i$. Implement the following SMO (Edwards & Spurgeon, 1994):

$$\dot{\hat{x}}^i = \tilde{A}^i\hat{x}^i - G_l^i e_y^i + G_n^i \nu^i, \quad e_y^i = \tilde{C}^i\hat{x}^i - y^i \qquad (10)$$

$\nu^i = -\rho^i \frac{e_y^i}{\|e_y^i\|}$, $G_n^i = \begin{bmatrix} -L^i \\ I_{p^i} \end{bmatrix}(P_o^i\tilde{C}_2^i)^{-1}$, $L^i = \begin{bmatrix} L_o^i & 0 \end{bmatrix}$ and L_o^i has $\breve{p}^i - \bar{p}^i$ columns, and G_n^i, L^i, P_o^i are designed so that a stable sliding motion takes place on $\mathcal{S}^i(e_y^i = \dot{e}_y^i = 0)$. To design those matrices, see Edwards and Spurgeon (1994) and Tan and Edwards (2001). It has been shown that sliding motion takes place if ρ^i is large enough (Tan & Edwards, 2003). When sliding motion has taken place; from the structure of L^i, $\tilde{M}_2^i$, $\tilde{Q}_2^i$ in (8), it is clear that $L^i\tilde{M}_2^i = 0$, $L^i\tilde{Q}_2^i = 0$. Define $v^i := x^i - \hat{x}^i$, $w^i := (P_o^i\tilde{C}_2^i)^{-1}\nu_{eq}^i$. Therefore the error dynamics in the coordinates of (8) can be partitioned into (see Tan & Edwards, 2003)

$$\dot{v}^i = (\tilde{A}_1^i + L^i\tilde{A}_3^i)v^i + \tilde{Q}_1^i\xi, \qquad w^i = \tilde{A}_3^i v^i + \tilde{Q}_2^i\xi + \tilde{M}_2^i f. \qquad (11)$$

(2) *Obtain system for next SMO*: Partition w^i using (9) to get

$$w_1^i = \tilde{A}_{31}^i v^i,$$

$$w_2^i = \begin{bmatrix} \tilde{A}_{32}^i \\ \tilde{A}_{33}^i \end{bmatrix} v^i + \begin{bmatrix} \tilde{Q}_{22}^i \\ 0 \end{bmatrix}\xi + \begin{bmatrix} 0 \\ M_o^i \end{bmatrix} f. \qquad (12)$$

Filter w_2^i to obtain z^i via $\dot{z}^i = -\alpha^i z^i + \alpha^i w_2^i$, $\alpha^i \in \mathbb{R}_+$. Substitute w_2^i from (12) and combine with (11) to obtain $\dot{x}^{i+1} = \tilde{A}^{i+1}x^{i+1} + \tilde{Q}^{i+1}\xi + \tilde{M}^{i+1}f$, $y^{i+1} = \tilde{C}^{i+1}x^{i+1}$ where $x^{i+1} := col\left\{v^i, z^i\right\}$, $y^{i+1} := col\left\{w_1^i, z^i\right\}$. Define $\tilde{Q}_{222}^{i+1} = \text{diag}\left\{Q_{22}^{i+1}, \alpha^i Q_{222}^i\right\}$, $\tilde{C}_2^{i+1} = \text{diag}\left\{Y^i, I_{\bar{k}^i+q}\right\}$ which are invertible. Using (5), (7) and (8) yields $\tilde{A}_1^{i+1} := \bar{A}_1^{i+1}$ and $\tilde{A}_3^{i+1}$, $\tilde{Q}_2^{i+1}$ respectively as

$$\begin{bmatrix} A_{31}^{i+1} & A_{32}^{i+1} \\ A_{33}^{i+1} & A_{34}^{i+1} \\ * & * \\ \alpha^i A_{35}^{i+1} & \alpha^i A_{36}^{i+1} \end{bmatrix}, \qquad \begin{bmatrix} 0 \\ \bar{Q}_{21}^{i+1} \\ \alpha^i\tilde{Q}_{22}^i \\ 0 \end{bmatrix} \begin{array}{l} \updownarrow \breve{p}^{i+1} - \bar{p}^{i+1} \\ \updownarrow k^{i+1} \\ \updownarrow \bar{k}^i \\ \updownarrow q. \end{array} \qquad (13)$$

By examining $\bar{Q}_{21}^i$ from (6) and $\tilde{Q}_{22}^i$ from (9), it can be seen that $\begin{bmatrix} \bar{Q}_{21}^{i+1} \\ \alpha^i\tilde{Q}_{22}^i \end{bmatrix} = \begin{bmatrix} 0 & Q_{22}^{i+1} & 0 \\ 0 & 0 & \alpha^i\tilde{Q}_{222}^i \end{bmatrix} = \begin{bmatrix} 0 & \tilde{Q}_{222}^{i+1} \end{bmatrix}$. Therefore $\tilde{Q}_2^i$ from (9) and $\tilde{Q}_2^{i+1}$ have the same structure. Since $\tilde{A}_1^i = \bar{A}_1^i$ and $\tilde{A}_{31}^i = \bar{A}_{31}^i$, then from $\bar{A}_1^i$ in (7) and $\bar{A}_{31}^i$ in (6), and by examining $\tilde{A}_1^i + L^i\tilde{A}_3^i$ it can be seen that $\tilde{A}^{i+1}$ has the same structure as $\tilde{A}^i$ in (8)–(9); hence the structures of $\tilde{A}^{i+1}$, $\tilde{M}^{i+1}$, $\tilde{C}^{i+1}$, $\tilde{Q}^{i+1}$ are consistent with $\tilde{A}^i$, $\tilde{M}^i$, $\tilde{C}^i$, $\tilde{Q}^i$ in (8)–(9).

(3) *Check for algorithm termination*: If $i + 1 \neq m$, increment i by 1 and return to Step 1; otherwise design observer m using the steps in Ng et al. (2010a) and terminate this algorithm. It can be seen that $(\tilde{A}^m, \tilde{M}^m, \tilde{Q}^m, \tilde{C}^m)$ from (13) have the same structure as (A, M, Q, C) in (1). Hence if A1 is satisfied for observer m, then the condition in Step 4a in the algorithm in Section 2.1 is satisfied, justifying the statement in that step.

Remark 1. It is clear that e_y^i vanishes when sliding motion occurs in observer i, which also causes the last p^i columns of $\tilde{A}^i$ to vanish from the analysis. This is because the 'output' states are the last p^i states, caused by the structure of $\tilde{C}^i$, which was in turn caused by the structure of $\bar{A}_{31}^i$ in (6). Since $\tilde{C}^{i+1}$ has the same structure as $\tilde{C}^i$, the last p^{i+1} columns of $\tilde{A}^{i+1}$ will also vanish during sliding motion of observer $i+1$. By expanding $\tilde{A}_1^i + L^i\tilde{A}_3^i$ using (5), and comparing with $\tilde{A}^{i+1}$ in (13), it can be seen that the elements of L^i appear only in the last p^{i+1} columns of $\tilde{A}^{i+1}$ and thus, L^i plays no role in the sliding motion of observer $i+1$, and does not affect the quality of the fault reconstruction. It is obvious that G_l^i also vanishes during sliding motion. Hence, in the design (G_l^i, L^i) do not affect the sliding motion of subsequent observers, and more importantly, does not affect the ability of the scheme to achieve DDFR. Furthermore, this means that each observer can be designed independently, thus reducing design complexity. □

Remark 2. In this paper, DDFR is possible even when the conditions in Ng et al. (2010a,b) are not satisfied. This is the main advantage of this paper. In the physical sense, this advantage can be translated into the fact that DDFR can potentially be achieved using fewer sensors, hence minimizing system complexity and reducing cost. This can be explained as follows: if there are fewer sensors, then p^1 is smaller and hence A_{32}^1 will have less rows, resulting in a lower chance of A1 being satisfied for 1 observer. A smaller p^1 potentially yields a smaller p^2 and A_{32}^2 will have less rows and again result in a lower chance of DDFR being achieved using 2 observers. However, the method in this paper goes beyond 2 observers and hence is potentially still capable of achieving DDFR even when it is not possible to do so with 1 or 2 observers.

Remark 3. The approach in this paper is similar to the 'step-by-step' method (Bejarano, Fridman, & Poznyak, 2007). As i increases, ν^i may cause excessive chattering or degradation in accuracy (Fridman, Davila, & Levant, 2008). This can be overcome by replacing ν^i with a 'supertwisting structure' (Tan & Edwards, 2010), which produces a smooth ν^i, and gives optimal performance at each step at least (Bejarano et al., 2007).

3. Existence conditions

It is possible to generate a disturbance decoupled fault reconstruction using m observers if the following are satisfied

B1. $\operatorname{rank}(A_{32}^m) = \operatorname{rank}\left[\begin{pmatrix}A_{12}^m\end{pmatrix}^T \quad \begin{pmatrix}A_{32}^m\end{pmatrix}^T \quad \begin{pmatrix}A_{36}^m\end{pmatrix}^T\right]^T$
B2. All observers have a stable sliding motion.

The remainder of this section investigates the limitation of this scheme, together with the conditions that guarantee B2.

Proposition 1. *If $\begin{bmatrix}A_{31}^i & A_{32}^i\end{bmatrix} = 0$ or non-existent, and if either A_{36}^i or A_{12}^i is nonzero, then DDFR can never be guaranteed.*

Proof. From Step 2 of the algorithm, if $\begin{bmatrix}A_{31}^i & A_{32}^i\end{bmatrix} = 0$ or does not exist, it results in $p^{i+1} = q + \bar{k}^i$ and $\bar{k}^i = \bar{k}^{i+1}$ (since $k^{i+1} = \operatorname{rank}(A_{32}^i)$) and then $p^{i+1} = q + \bar{k}^{i+1}$. Then both A_{31}^{i+1} and A_{12}^{i+1} from (13) will not exist since they have row dimensions of $p^{i+1} - q - \bar{k}^{i+1}$, and neither would $A_{31}^j, A_{32}^j, j = i+1, i+2, \ldots$. Also, when $\begin{bmatrix}A_{31}^i & A_{32}^i\end{bmatrix} = 0$, then $T^i = I_{n^i}$ and $A_{36}^i = A_{36}^{i+1}, A_{12}^i = A_{12}^{i+1}$ and either A_{36}^j, A_{12}^j $(j = i+1, i+2, \ldots)$ will be non-zero, B1 will never be satisfied and DDFR can never be guaranteed. □

Proposition 2. *If observer $n-p$ cannot guarantee DDFR, then neither can the subsequent observers.*

Proof. It was established that $p^{i+1} \le p^i$ and $\bar{k}^{i+1} \ge \bar{k}^i$ resulting in (see (3)) $\dim(Y^i) \ge \dim(Y^{i+1}) > 0$; by analyzing the column dimension of $\bar{A}$ in (3) it can be shown that the maximum value for i is $n-p-1$ and therefore $\begin{bmatrix}A_{31}^{n-p} & A_2^{n-p}\end{bmatrix}$ has 1 column at most. If A_{31}^{n-p} spans that column, then A_{32}^{n-p} does not exist and $h - \bar{k}^{n-p} = 0$ and from Ng et al. (2010a) observer $n-p$ can reconstruct ξ and achieve DDFR. However, if A_{32}^{n-p} spans the single column, then two possibilities are to be considered: if $A_{32}^{n-p} \ne 0$, then B1 is satisfied and DDFR can be achieved with observer $n-p$; however if $A_{32}^{n-p} = 0$ or does not exist and B1 is not satisfied, then from Proposition 1, DDFR can never be guaranteed. □

Theorem 1. *All m observers will have a stable sliding motion if $\left(A^1, \begin{bmatrix}M^1 & Q^1\end{bmatrix}, C^1\right)$ is minimum phase.*

The remainder of this section will prove Theorem 1. From (11), the sliding motion for each observer is governed by $\tilde{A}_1^i + L_o^i\tilde{A}_{31}^i$. Hence $(\tilde{A}_1^i, \tilde{A}_{31}^i)$ from (8) must be detectable. □

Proposition 3. *Partition $\tilde{Q}^i$ from (8) as $\tilde{Q}^i = \begin{bmatrix}\tilde{Q}_a^i & \tilde{Q}_b^i\end{bmatrix}$ where $\tilde{Q}_b^i$ has $\bar{k}^i$ columns. Then the unobservable modes of $(\tilde{A}_1^i, \tilde{A}_{31}^i)$ are the zeros of $\left(\tilde{A}^i, \begin{bmatrix}\tilde{Q}_b^i & \tilde{M}^i\end{bmatrix}, \tilde{C}^i\right)$.*

Proof. Define $\mathcal{P}_1(s)$ as the Rosenbrock matrix (Rosenbrock, 1970) of $\left(\tilde{A}^i, \begin{bmatrix}\tilde{Q}_b^i & \tilde{M}^i\end{bmatrix}, \tilde{C}^i\right)$; the zeros of the system are the values of s that cause its Rosenbrock matrix to lose rank. Substitute for $\tilde{A}^i, \tilde{Q}_b^i, \tilde{M}^i, \tilde{C}^i$ from (8)–(9) into $\mathcal{P}_1(s)$. Since $\tilde{Q}_{222}^i, M_o^i$ and $\tilde{C}_2^i$ are invertible, then by using the Popov–Hautus–Rosenbrock (PHR) rank test (Rosenbrock, 1970) $\mathcal{P}_1(s)$ loses rank when s is an unobservable mode of $(\tilde{A}_1^i, \tilde{A}_{31}^i)$ (see Proposition 6 of Tan & Edwards, 2010). □

Proposition 4. *The unobservable modes (if any) of observer i will also be the unobservable modes of observer $i+1$.*

Proof. From Proposition 3, the unobservable modes of observer i are the values of s that make $\mathcal{P}_2(s)$ lose rank.

Substitute for $\tilde{A}_1^i$ and $\tilde{A}_{31}^i$ (from (8)–(9)) into $\mathcal{P}_2(s)$. Knowing that $\tilde{A}_{31}^i = \bar{A}_{31}^i$ and from the structure of $\bar{A}_{31}^i$ in (6), the values of s that make $\mathcal{P}_2(s)$ lose rank are the unobservable modes of $\left(\bar{A}_1^{i+1}, \left[\left(\bar{A}_{31}^{i+1}\right)^T \quad \left(\bar{A}_{32}^{i+1}\right)^T\right]^T\right)$.

From Proposition 3, the unobservable modes of observer $i+1$ are the zeros of $\left(\tilde{A}^{i+1}, \begin{bmatrix}\tilde{Q}_b^{i+1} & \tilde{M}^{i+1}\end{bmatrix}, \tilde{C}^{i+1}\right)$. Let the Rosenbrock matrix of $\left(\tilde{A}^{i+1}, \begin{bmatrix}\tilde{Q}_b^{i+1} & \tilde{M}^{i+1}\end{bmatrix}, \tilde{C}^{i+1}\right)$ be $\mathcal{P}_3(s)$ and substitute from (13). Since $Q_{22}^{i+1}, \tilde{Q}_{222}^i, M_o^i$ and Y^i are invertible, then the values of s that make $\mathcal{P}_3(s)$ lose rank are the unobservable modes of $(\bar{A}_1^{i+1}, \bar{A}_{31}^{i+1})$. Hence the values of s that make $\mathcal{P}_2(s)$ lose rank also make $\mathcal{P}_3(s)$ lose rank; and an unobservable mode of observer i will also be an unobservable mode of observer $i+1$. □

Proposition 5. *If $\left(A, \begin{bmatrix}M & Q\end{bmatrix}, C\right)$ is minimum phase, then $\left(\tilde{A}^m, \begin{bmatrix}\tilde{M}^m & \tilde{Q}_b^m\end{bmatrix}, \tilde{C}^m\right)$ is also minimum phase.*

Proof. Let the Rosenbrock matrix of $\left(A, \begin{bmatrix}M & Q\end{bmatrix}, C\right)$ be $\mathcal{P}_4(s)$. Substituting for $(\bar{A}, \bar{M}, \bar{Q}, \bar{C})$ (which are transformations of (A, M, Q, C)) from (3), (4) and (2) into $\mathcal{P}_4(s)$, and knowing that $M_o, C_2, Q_{22}^m \ldots Q_{22}^1, Y^{m-1} \ldots Y^1$ and Q_{11}^m are full rank, then $\mathcal{P}_4(s)$ loses rank when s is a zero of $(A_{11}^m, A_{12}^m, A_{31}^m, A_{32}^m)$. Then let $\mathcal{P}_5(s)$ be the Rosenbrock matrix of $\left(\tilde{A}^m, \begin{bmatrix}\tilde{M}^m & \tilde{Q}^m\end{bmatrix}, \tilde{C}^m\right)$. Substitute from (13) into $\mathcal{P}_5(s)$; and $\mathcal{P}_5(s)$ will lose rank when s is a zero of $(A_{11}^m, A_{12}^m, A_{31}^m, A_{32}^m)$. Hence $\left(\tilde{A}^m, \begin{bmatrix}\tilde{M}^m & \tilde{Q}^m\end{bmatrix}, \tilde{C}^m\right)$ and $\left(A, \begin{bmatrix}M & Q\end{bmatrix}, C\right)$ have the same zeros.

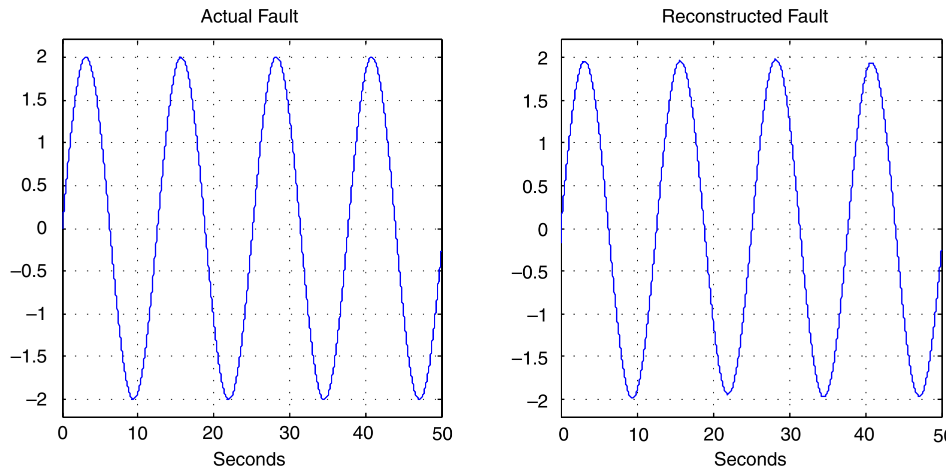

Fig. 1. The left subfigure is f, the right subfigure is $\hat{f}$.

Then define $\mathcal{R}_1(s)$ and $\mathcal{R}_2(s)$ as the Rosenbrock matrices of $\left(\tilde{A}^m, \left[\tilde{M}^m \quad \tilde{Q}_b^m\right], \tilde{C}^m\right)$ and $\left(\tilde{A}^m, \left[\tilde{M}^m \quad \tilde{Q}^m\right], \tilde{C}^m\right)$ respectively. It can be seen that if $\mathcal{R}_1(s)$ loses rank then $\mathcal{R}_2(s)$ also loses rank, but the converse is not necessarily true. As a result, the zeros of $\left(\tilde{A}^m, \left[\tilde{M}^m \quad \tilde{Q}_b^m\right], \tilde{C}^m\right)$ will also be the zeros of $\left(\tilde{A}^m, \left[\tilde{M}^m \quad \tilde{Q}^m\right], \tilde{C}^m\right)$. $\square$

Combining Propositions 3 and 5 means observer m will be detectable if $(A, \left[M \quad Q\right], C)$ is minimum phase, since $\tilde{Q}^i = \left[\tilde{Q}_a^i \quad \tilde{Q}_b^i\right]$. Then Proposition 4 further implies that all previous observers are also detectable. Therefore, if $\left(A, \left[M \quad Q\right], C\right)$ is minimum phase, then all observers have a stable sliding motion and Theorem 1 is proven. $\square$

4. Simulation example

A 3rd order water tank is used as an example, described by $\dddot{\theta} + 10\ddot{\theta} + 31\dot{\theta} + 30\theta = u$ where θ and u are the water height and inlet flow rate, respectively, which are measurable. Suppose the sensor for u is faulty. Denote θ_m, u_m to be the measurements for θ and u, respectively, hence $\theta_m = \theta$, $u_m = u + f$, where f encapsulates potential faults. Then u_m is filtered to obtain z_f via $\dot{z}_f = -z_f + u_m$. Suppose u is generated by a first order nonlinear device via $\dot{u} = -u + u_c + \xi$ where u_c is the flow command and ξ is the nonlinearity. Define the states and outputs respectively as $x = [\ddot{\theta}\ \dot{\theta}\ u\ \theta z_f]^T$ and $y = [\theta_m\ z_f]^T$; hence the following respective state space matrices A, M, Q, C are obtained:

$$\left[\begin{array}{ccc|cc} -10 & -31 & 1 & -30 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \end{array}\right], \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \left[\begin{array}{c} 0 \\ 0 \\ 1 \\ \hline 0 \\ 0 \end{array}\right], \quad \left[\begin{array}{cc} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \hline 1 & 0 \\ 0 & 1 \end{array}\right]^T. \tag{14}$$

Notice that Q, M, C in (14) are already in the form of (1)–(2), with $n = 5, p = 2, q = 1, h = 1$. It is obvious that $\text{rank}\left(C\left[M \quad Q\right]\right) < \text{rank}(M) + \text{rank}(Q)$ and linear observers (Saif & Guan, 1993) cannot achieve DDFR.

4.1. Algorithm to calculate number of observers

It is obvious that $\text{rank}(CM) = \text{rank}(M)$ and that $\text{rank}(C\left[M \quad Q\right]) = \text{rank}(CM) + \text{rank}(CQ)$, $CQ = 0 \Rightarrow \bar{k} = 0$. By comparing (14) with (1), A1 is not satisfied and therefore DDFR is impossible using 1 observer. The algorithm in Section 2.1 will now be entered:

Iteration $i = 1$: It follows that $k^2 = \text{rank}(A_{32}^1) = 0$, $p^2 = 2$, hence $\bar{k}^2 = 0$. It can be shown that $\bar{T}^1$ in (15) places the matrices in (14) in the structure of (3). The termination conditions in Steps 4a–4b are not satisfied (DDFR cannot be guaranteed with 2 observers), and the algorithm is repeated.

Iteration $i = 2$: It follows that $p^3 = 2$ and $k^3 = 0$, and can be shown that $\bar{T}^2$ in (15) places A, Q in the structure of (3)–(4) which satisfies the condition in Step 4a for $i = 2$, $m = 3$, and the algorithm is terminated. Hence DDFR can be achieved using 3 observers.

$$\bar{T}^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & I_2 \end{bmatrix}, \qquad \bar{T}^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & I_3 \end{bmatrix}. \tag{15}$$

It is verified that $(A^1, [M^1 Q^1], C^1)$ has no zeros and (as in Theorem 1), all observers can have a stable sliding motion.

4.2. Observer design and simulation results

The algorithm in Section 2.2 is then used to design the observers, as follows:

Observer 1: The matrix L_o^1 is chosen so that $\lambda(\tilde{A}_1^1 + L^1\tilde{A}_{31}^1) = \{-3, -10, -1\}$. Then, the matrices $P_o^1 = I_2$,

$$G_l^1 = \begin{bmatrix} -0.6932 & -30.7920 & -6.8620 & 9.9720 & -0.3597 \\ 7.1049 & 14.2021 & -1.3411 & -0.7507 & 8.0280 \end{bmatrix}^T$$

guarantee sliding motion. Choosing $\alpha^1 = 1$ results in $\tilde{A}^2, \tilde{M}^2, \tilde{Q}^2, \tilde{C}^2$; for which DDFR cannot be achieved (Ng et al., 2010a), hence the method in (Ng et al., 2010b) cannot achieve DDFR.

Observer 2: The matrix L_o^2 is chosen such that $\lambda(\tilde{A}_1^2 + L^2\tilde{A}_3^2) = \{-13, -1\}$. The choices

$P_o^2 = I_2, G_l^2 = \begin{bmatrix} 0.3962 & 12.0137 & -0.9725 & 0.1863 \\ 4.9540 & -0.5780 & 0.5616 & 5.9725 \end{bmatrix}^T$

guarantee sliding motion. Choosing $\alpha^2 = 1$ results in $\tilde{A}^3, \tilde{Q}^3, M^3, C^3$ for which it was found (from Ng et al., 2010a) that DDFR can be achieved and this confirms the result from Section 4.1 that DDFR can be achieved with 3 observers.

Observer 3: As DDFR conditions are already satisfied, the 3rd observer is then designed using the method in Ng et al. (2010a) where L_o^3 is chosen such that $\lambda(\tilde{A}_1^3 + L^3\tilde{A}_3^3) = -4$, together with $P_o^3 = I_2$, $G_l^3 = \begin{bmatrix} 2.7045 & -7.3009 & 1.5520 \\ 2.2925 & 1.4223 & 4.3009 \end{bmatrix}^T$. Then, choose $W_1^3 = -1$ to get $W^3 = \begin{bmatrix} -1 & 1 \end{bmatrix}$.

A fault is injected into the sensor of u together with the disturbance ξ which is assumed to be a sine wave. The left subfigure of Fig. 1 shows the actual fault while the right subfigure shows the fault reconstruction, which is identical to the fault despite ξ, thus confirming that DDFR is achieved.

As shown in Section 4.2, it is not possible to achieve DDFR using 1 or 2 observers, but it required 3 observers. If $\dot{\theta}$ is available, DDFR is possible using 2 observers and if $\ddot{\theta}$ is available, then DDFR would be possible using only 1 observer. However, in this paper, DDFR was achieved without the sensors for $\dot{\theta}$ and $\ddot{\theta}$; hence one of the main advantages is that DDFR can be attained with potentially fewer sensors.

5. Conclusion

This paper presented a DDFR scheme using cascaded SMOs. Signals from an observer are treated as the output of a 'fictitious' system, and another observer is designed for the fictitious system; this process is repeated until a system that satisfies DDFR conditions is obtained. The faults are reconstructed using the final observer. It was found that DDFR is achievable for a wider class of systems, specifically for system with fewer sensors, compared to situations if only one or two observers are being used. The paper also presented an algorithm that calculates the number of SMOs for DDFR, and an algorithm to design the SMOs. A third order system with actuator dynamics was used to validate the theory.

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